Lett. Math. Phys. in press
YITP/K-1112
q-alg/9506019

" EIGENVALUES OF CASIMIR INVARIANTS FOR TYPE I QUANTUM SUPERALGEBRAS "

by

Mark D. Gould, Jon R. Links

Department of Mathematics, University of Queensland, Queensland, 4072, Australia

and

Yao-Zhong Zhang

Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606, Japan

March, 1995

We present the eigenvalues of the Casimir invariants for the type I quantum superalgebras on any irreducible highest weight module.

Mathematics Subject Classifications 81R10, 17B37, 16W30

1. Introduction

Quantum algebras [1] are well known for their role in solving the Yang-Baxter equation and representing the symmetries of the associated solvable models. Likewise, their $\mathbb{Z}_2$-graded counterparts quantum superalgebras [2, 3, 4] play a similar role in relation to supersymmetric solvable models. There has been significant interest in the study of these models, particularly those describing systems of correlated electrons which are of importance in condensed matter physics. Such examples are the supersymmetric t-j model [5] and its quantum analogue [6], the supersymmetric extended Hubbard model [7] and the model of Bracken et. al. [8]. All of these models are exactly solvable in one dimension.

In the investigation of any model possessing a quantum (super)algebra as a symmetry, it is desirable to have a well developed representation theory. An important part in this pursuit is to study the central elements or Casimir invariants. There are established techniques for the construction of Casimir invariants, both for quantum algebras [9] and quantum superalgebras [10, 11]. Here we follow the construction of [10] where the Casimir invariants are obtained from tensor operators derived from the universal R-matrix and an *arbitrary* reference representation. Such general invariants are important in the computation of link invariants in knot theory [12]. However a formula for the eigenvalues of these invariants when acting on finite dimensional irreducible representations has never been derived. In this Letter we will present such a formula for the type I quantum superalgebras consisting of $U_q(gl(m|n))$ and $U_q(osp(2|2n))$. The proof of the formula is both lengthy and detailed and will be deferred to a separate publication. Here we will merely report our results which are new even in the classical case ($q \to 1$ limit). Our results are in complete agreement with studies of some particular cases [13, 14], but are more general and widely applicable.

Although our results apply only for the type I quantum superalgebras, they are the most interesting for the following reasons. Firstly, they admit finite dimensional unitary representations [15] which have physical importance where unitarity is a requirement (e.g. see [8]). Secondly, the type I quantum superalgebras admit one parameter families of representations which have interesting applications such as providing solutions to the Yang-Baxter equation with extra spectral parameters, though in a non-additive form [16]. Another application is in the construction of

two variable link invariants as indicated in [17]. However, there have always been technical difficulties in evaluating such link invariants. The results reported in the present Letter now permit a unified construction and evaluation of these invariants [18].

2. Preliminaries

Let g denote a basic classical Lie superalgebra of rank $l+1$ with the usual generators $\{e_i,\, f_i,\, h_i\}_{i=0}^{l}$. Let $\{\alpha_i\}_{i=0}^{l}$ be the distinguished set of simple roots of g in the sense of Kac [19] and let $(\ ,\ )$ be a fixed invariant bilinear form on H^*, the dual of the Cartan subalgebra H of g. We also let $\Phi^+ = \Phi_0^+ \cup \Phi_1^+$ denote the full set of positive roots with Φ_0^+ (resp. Φ_1^+) the subset of even (resp. odd) positive roots. Throughout, we adopt the convention that α_0 denotes the unique odd simple root. Associated with g one can define the quantum superalgebra $U_q(g)$ (q is assumed not a root of unity) which has the structure of a $\mathbb{Z}_2$-graded quasi-triangular Hopf algebra [3]. We will not give the full defining relations of $U_q(g)$ here and refer to [4] for details. We note however that $U_q(g)$ has a co-product structure given by

$$\Delta(q^{\pm\frac{1}{2}h_i}) = q^{\pm\frac{1}{2}h_i} \otimes q^{\pm\frac{1}{2}h_i}, \quad \Delta(x) = x \otimes q^{-\frac{1}{2}h_i} + q^{\frac{1}{2}h_i} \otimes x, \quad x = e_i,\, f_i$$

which is extended to an algebra homomorphism to all of $U_q(g)$ in the usual way. It is important to point out that the multiplication rule for the tensor product is defined for homogeneous elements $a, b, c, d \in U_q(g)$ by

$$(a \otimes b)(c \otimes d) = (-1)^{[b][c]}(ac \otimes bd) \tag{1}$$

and extended linearly to all of $U_q(g) \otimes U_q(g)$. Here $[a] \in \mathbb{Z}_2$ denotes the degree of the homogeneous element $a \in U_q(g)$, which is defined for the elementary generators by

$$[h_i] = 0, \quad [e_i] = [f_i] \equiv [i] = \delta_{i0}, \qquad \forall\, 0 \le i \le l,$$

and extended to all homogeneous elements of $U_q(g)$ through

$$[ab] = [a] + [b] (mod\, 2), \qquad \forall\, a, b \in U_q(g).$$

The twist map $T\colon U_q(g) \otimes U_q(g) \to U_q(g) \otimes U_q(g)$ is defined by

$$T(a \otimes b) = (-1)^{[a][b]} b \otimes a \tag{2}$$

for all homogeneous $a, b \in U_q(g)$: we set $\overline{\Delta} = T.\Delta$. Then there exists a canonical element $R \in U_q(g) \otimes U_q(g)$ called the universal R-matrix which is even and invertible and satisfies the following relations [3]

$$R\Delta(a) = \overline{\Delta}(a)R, \qquad \forall a \in U_q(g), \tag{3}$$

$$(\Delta \otimes I)R = R_{13}R_{23}, \quad (I \otimes \Delta)R = R_{13}R_{12}, \tag{4}$$

where we have adopted the conventional notation. From equations (3) and (4) it follows that the universal R-matrix satisfies the graded Yang-Baxter equation

$$R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12}. \tag{5}$$

We emphasize that multiplication of the tensor products are to obey equation (1).

Let $\rho \in H^*$ denote the graded half sum of positive roots of g and let h_ρ denote the unique element of H defined by $\alpha_i(h_\rho) = (\rho, \alpha_i), \quad \forall \alpha_i \in H^*$. We recall from [10] the following result.

Theorem 1. *Let π be a fixed, but arbitrary, finite dimensional representation of $U_q(g)$ with representation space V and set*

$$\Delta_\pi = (\pi \otimes I)\Delta.$$

If $w \in \mathit{End}\, V \otimes U_q(g)$ satisfies

$$\Delta_\pi(a)w = w\Delta_\pi(a), \qquad \forall a \in U_q(g) \tag{6}$$

then

$$s\tau_q(w) = (str \otimes I)(\pi(q^{2h_\rho}) \otimes I)w$$

belongs to the centre of $U_q(g)$, where str denotes the supertrace.

Theorem 1 enables a family of Casimir invariants to be constructed for $U_q(g)$ for any reference module V utilizing the universal R-matrix. Defining $R^T = TR$, it is clear from (3) that

$$R^T R\Delta(a) = \Delta(a)R^T R, \qquad \forall a \in U_q(g).$$

Setting

$$A = (q - q^{-1})^{-1}(\pi \otimes I)(I \otimes I - R^T R)$$

then A^l, $l \in \mathbb{Z}^+$, satisfies (6). We thus obtain the family of Casimir invariants

$$C_l = s\tau_q(A^l). \tag{7}$$

In the limit $q \to 1$ these give rise to a family of Casimir invariants for the classical Lie superalgebra g.

The preceeding discussion applies for any quantum superalgebra, not only those of type I. Let $V(\mu)$ denote a finite dimensional irreducible $U_q(g)$ module of highest weight $\mu \in D^+$ where $D^+ \subset H^*$ is the set of dominant weights. For type I quantum superalgebras, $\mu \in D^+$ if and only if [19, 20]

$$< \mu, \alpha_i >= \frac{2(\mu,\, \alpha_i)}{(\alpha_i,\, \alpha_i)} \in \mathbb{Z}^+, \qquad 1 \leq i \leq l,$$

while $(\mu,\, \alpha_0)$ can take arbitrary complex values. When acting on $V(\mu)$ the invariants C_l act as scalar multiples of the identity operator (Schur's lemma), which we denote by $\chi_\mu(C_l)$. A general formula for these eigenvalues is unknown. In the next section we will present such a formula for the type I quantum superalgebras.

3. Eigenvalue Formula

Hereafter $U_q(g)$ is assumed to be of type I. Recall that for type I quantum superalgebras we say that $\mu \in H^*$ is *typical* if [19, 20]

$$(\mu + \rho,\, \alpha) \neq 0, \qquad \forall \alpha \in \Phi_1^+,$$

and *atypical* otherwise. We define the following.

Definition 1. *Let $\Pi(\Lambda)$ denote the weight spectrum of $V(\Lambda)$. We say that Λ is subordinate to $\mu \in D^+$ if $\forall \lambda \in \Pi(\Lambda)$, $\mu + \lambda$ is dominant. Moreover we say that Λ is typically subordinate to μ if μ, $\mu + \lambda$ are all typical and dominant. We denote the set of such μ by D_Λ^+.*

Let λ_i denote the distinct weights in the reference module $V(\Lambda)$ with $[\lambda_i]$ the degree of λ_i. For $\mu \in D_\Lambda^+$ we have the tensor product decomposition

$$V(\Lambda) \otimes V(\mu) = \bigoplus_i m_i V(\mu + \lambda_i), \tag{8}$$

where m_i denotes the multiplicity of $V(\mu + \lambda_i)$. The above decomposition is necessarily completely reducible since any typical module splits in a finite dimensional

representation [19]. Also m_i is the same as the multiplicity of λ_i occurring in $\Pi(\Lambda)$ (see lemma A2 in the appendix). If

$$P[i] \equiv P[\mu + \lambda_i]$$

denote central projections onto the isotypic components $\overline{V(\mu+\lambda_i)} \equiv m_i V(\mu+\lambda_i)$ we have the spectral decomposition [10]

$$A^l = \sum_i [\beta_i(\mu)]^l \, P[i],$$

where the roots $\beta_i(\mu)$ are given by

$$\beta_i(\mu) = \frac{1 - q^{-(\lambda_i, \lambda_i + 2\mu + 2\rho) + (\Lambda, \Lambda + 2\rho)}}{q - q^{-1}}.$$

Since the $P[i]$ are central projections, we may invoke theorem 1 to construct the central elements

$$\gamma[i] = s\tau_q(P[i]),$$

which leads to

$$\chi_\mu(C_l) = \sum_i [\beta_i(\mu)]^l \, \chi_\mu\left(\gamma[i]\right).$$

Recently, we have determined the quantities $\chi_\mu\left(\gamma[i]\right)$ [21]. The details of the derivation of our results are beyond the scope of this Letter. Here we merely wish to present our results.

Proposition 1. *For any $\overline{V(\mu+\lambda_i)} \subset V(\Lambda) \otimes V(\mu)$, $\lambda_i \in \Pi(\Lambda)$ such that μ, $\mu+\lambda_i$ are both typical and dominant then*

$$\chi_\mu\left(\gamma[i]\right) = (-1)^{[\lambda_i]} m_i \prod_{\alpha \in \Phi_0^+} \frac{[(\mu + \lambda_i + \rho, \, \alpha)]_q}{[(\mu + \rho, \, \alpha)]_q} \prod_{\alpha \in \Phi_1^+} \frac{[(\mu + \rho, \, \alpha)]_q}{[(\mu + \lambda_i + \rho, \, \alpha)]_q},$$

where

$$[x]_q = \frac{q^x - q^{-x}}{q - q^{-1}}.$$

With the aid of proposition 1, we can determine the eigenvalues of the Casimir invariants C_l on all irreducible finite dimensional modules $V(\mu)$ such that Λ is typically subordinate to μ. In view of the following, we can extend our formula to all modules $V(\mu)$.

Proposition 2. *Let $\Lambda \in D^+$ be fixed but arbitrary. If f is a polynomial function on H^* satisfying $f(\nu) = 0, \quad \forall \nu \in D^+_\Lambda$, then f vanishes identically.*

The proof of proposition 2 is left to the appendix.

Consider the quantity

$$f(\mu) = \chi_\mu(C_l) - \sum_i [\beta_i(\mu)]^l \, \chi_\mu(\gamma[i]),$$

with $\chi_\mu(\gamma[i])$ as in proposition 1. It is apparent that

$$f(\mu) = 0, \qquad \forall \mu \in D^+_\Lambda.$$

Expanding $f(\mu)$ into a power series in $\eta = ln\, q$ we obtain

$$f(\mu) = \sum_k f_k(\mu)\eta^k,$$

where $f_k(\mu)$ is a polynomial function on H^*. It follows that

$$f_k(\mu) = 0, \qquad \forall\, \mu \in D^+_\Lambda.$$

In view of proposition 2 we have

$$f_k(\mu) = 0, \qquad \forall\, \mu \in H^*,$$

from which we deduce the following.

Proposition 3. *The eigenvalues of the Casimir invariants (7) acting on the irreducible module $V(\mu), \quad \forall \mu \in D^+$, are given by*

$$\chi_\mu(C_l) = \sum_i (-1)^{[\lambda_i]} m_i \, [\beta_i(\mu)]^l \prod_{\alpha \in \Phi_0^+} \frac{[(\mu + \lambda_i + \rho,\, \alpha)]_q}{[(\mu + \rho,\, \alpha)]_q} \prod_{\alpha \in \Phi_1^+} \frac{[(\mu + \rho,\, \alpha)]_q}{[(\mu + \lambda_i + \rho,\, \alpha)]_q}. \tag{9}$$

The above eigenvalue formula in fact holds on any $U_q(g)$ module admitting an infinitesimal character χ_μ, $\mu \in H^*$. However for atypical $\mu + \lambda_i$, proposition 3 as presented is undefined. Nevertheless, it is important to observe that $\chi_\mu(C_l)$ is in fact a polynomial function in $q^{\pm(\mu,\, \alpha_i)}$, $i = 0, 1, ..., l$. Hence in principle eq. (9) may be expanded to yield a well defined expression.

In the limit $q \to 1$ we obtain the following result for the classical Lie superalgebra case:

$$\chi_\mu(C_l) = \sum_i (-1)^{[\lambda_i]} \, [\beta_i(\mu)]^l \prod_{\alpha \in \Phi_0^+} \frac{(\mu + \lambda_i + \rho,\, \alpha)}{(\mu + \rho,\, \alpha)} \prod_{\alpha \in \Phi_1^+} \frac{(\mu + \rho,\, \alpha)}{(\mu + \lambda_i + \rho,\, \alpha)},$$

where

$$\beta_i(\mu) = \frac{1}{2}(\lambda_i,\, \lambda_i + 2\mu + 2\rho) - \frac{1}{2}(\Lambda,\, \Lambda + 2\rho).$$

This result is also new. It generalizes to arbitrary reference representations the results of [14].

4. Examples

We will now illustrate our results for the case when Λ is the vector representation. Let us first consider $U_q(gl(m|n))$. We choose $\{\varepsilon_i\}_{i=1}^{m+n}$ as a basis for H^* with the $\mathbb{Z}_2$-gradation

$$[\varepsilon_i] \equiv [i] = \begin{cases} 0, & 1 \le i \le m, \\ 1, & m < i \le m+n, \end{cases}$$

and equipped with the invariant bilinear form

$$(\varepsilon_i,\, \varepsilon_j) = (-1)^{[i]}\delta_{ij}.$$

The sets of even and odd positive roots are respectively given by

$$\Phi_0^+ = \{\varepsilon_i - \varepsilon_j | 1 \le i < j \le m+n,\, [i] = [j]\},$$
$$\Phi_1^+ = \{\varepsilon_i - \varepsilon_j | 1 \le i < j \le m+n,\, [i] \ne [j]\},$$

in terms of which the graded half sum of positive roots is expressed as

$$\rho = \frac{1}{2}\sum_{i=1}^{m}(m - n - 2i + 1)\varepsilon_i - \frac{1}{2}\sum_{j=1}^{n}(m + n - 2j + 1)\varepsilon_{m+j}.$$

The vector representation of $U_q(gl(m|n))$ has highest weight $\Lambda = \varepsilon_1$ and the full weight spectrum is $\Pi(\varepsilon_1) = \{\varepsilon_i\}_{i=1}^{m+n}$. Applying formula (9) for the eigenvalues of the Casimir invariants (7) (with $\Lambda = \varepsilon_1$) acting on the module $V(\mu)$ yields

$$\chi_\mu(C_l) = \sum_{i=1}^{m+n}(-1)^{[i]}\,[\beta_i(\mu)]^l \prod_{j\ne i}^{m+n} \frac{q^{-(\varepsilon_j,\varepsilon_j)}\beta_i(\mu) - q^{(\varepsilon_j,\varepsilon_j)}\beta_j(\mu) + (-1)^{[j]}}{\beta_i(\mu) - \beta_j(\mu)}$$

where the roots $\beta_i(\mu)$ are given by

$$\beta_i(\mu) = \frac{1 - q^{-(\varepsilon_i,\varepsilon_i+2\mu+2\rho)+m-n}}{q - q^{-1}}.$$

The above eigenvalue formula is in agreement with the results of [13] obtained by different means. In the $q \to 1$ limit the above formula coincides with those of [14] taking into account the different conventions adopted.

Next we consider the case $U_q(osp(2|2n))$. We choose the basis $\{\varepsilon_i\}_{i=0}^n$ for H^* with the invariant bilinear form

$$(\varepsilon_i,\, \varepsilon_j) = (-1)^{[i]}\delta_{ij},$$

where ε_0 is the unique odd basis element. The sets of even and odd positive roots are respectively given by

$$\begin{aligned}\Phi_0^+ &= \{\varepsilon_i \pm \varepsilon_j | 1 \le i < j \le n\} \cup \{2\varepsilon_i | 1 \le i \le n\},\\ \Phi_1^+ &= \{\varepsilon_0 \pm \varepsilon_i | 1 \le i \le n\},\end{aligned}$$

while the graded half sum of positive roots reads

$$\rho = -n\varepsilon_0 + \sum_{i=1}^{n}(n-i+1)\varepsilon_i.$$

The vector representation of $U_q(osp(2|2n))$ has highest weight $\Lambda = \varepsilon_0$ and weight spectrum $\Pi(\varepsilon_0) = \{\pm\varepsilon_i\}_{i=0}^n$. Applying the eigenvalue formula (9) yields

$$\begin{aligned}\chi_\mu(C_l) = \sum_{i=-n}^{n} (-1)^{[i]}\, [\beta_i(\mu)]^l\, &\frac{q^{-1-(\varepsilon_i,\varepsilon_i)}\beta_i(\mu) - q^{1+(\varepsilon_i,\varepsilon_i)}\beta_{-i}(\mu) + q + (-1)^{[i]} q^{-(\varepsilon_i,\varepsilon_i)}}{\beta_i(\mu) - \beta_{-i}(\mu)}\\ \times \prod_{j \ne \pm i} &\frac{q^{-(\varepsilon_j,\varepsilon_j)}\beta_i(\mu) - q^{(\varepsilon_j,\varepsilon_j)}\beta_j(\mu) + (-1)^{[j]}}{\beta_i(\mu) - \beta_j(\mu)}\end{aligned}$$

where

$$\beta_i(\mu) = \frac{1 - q^{-(\varepsilon_i, \varepsilon_i + 2\mu + 2\rho) + 2n - 1}}{q - q^{-1}}.$$

In the above formula it should be understood that

$$\varepsilon_{-i} \equiv -\varepsilon_i, \qquad [-i] \equiv [i].$$

For generic values of q the above eigenvalue formula is a new result. As $q \to 1$ it agrees with the formulae presented in [14].

5. Conclusion

In this Letter we have presented a general eigenvalue formula for the Casimir invariants of the type I quantum superalgebras when acting on irreducible finite dimensional modules. Our formula applies to arbitrary reference modules and the eigenvalues are expressed in terms of the highest weight labels. The construction of these Casimir invariants is given in an earlier publication [10]. Our results are new even in the classical $q \to 1$ limit and will have application in a variety of areas including the evaluation of link invariants [18]. As examples we have explicitly evaluated these eigenvalues in the case when the reference module is the vector module. For the case of $U_q(gl(m|n))$ our results agree with those of [13] obtained by an entirely different method, while for $U_q(osp(2|2n))$ our results are new. In the limit $q \to 1$ our results are also in agreement with [14] taking into account the different conventions adopted.

Acknowledgement.

This research is supported by the Australian Research Council. YZZ is financially supported by the Kyoto University Foundation.

Appendix

Here we will prove proposition 2. To do so, we will require a few technical results. The first is due to Kostant [22].

Lemma A1. *Impose a partial ordering on H^* with the order relation $\mu \geq \nu$ if $\mu - \nu \in D^+$. Let $\nu_0 \in D^+$ be arbitrary and let f be any polynomial function on H^*. If $f(\nu) = 0$ for all $\nu \geq \nu_0$ then f vanishes identically on H^*.*

Lemma A1 is proved in [22] for the case of non $\mathbb{Z}_2$-graded algebras, but is easily extended to the superalgebra case. We will also require the following [21].

Lemma A2. *Let $U_q(g_0) \subset U_q(g)$ be the (non-graded) quantum algebra generated by $\{e_i, f_i, h_i\}_{i=1}^l$. If*

$$V(\Lambda) \otimes V(\mu) = \bigoplus_i m_i V(\nu_i),$$

where μ, ν_i are all typical and dominant then we have the $U_q(g_0)$ decomposition

$$V(\Lambda) \otimes V_0(\mu) = \bigoplus m_i V_0(\nu_i),$$

where $V_0(\nu)$ denotes the $U_q(g_0)$ module of highest weight ν and $V(\Lambda)$ is a direct sum of such modules; viz.

$$V(\Lambda) = \bigoplus_j V_0(\Lambda_j).$$

In the case that Λ is subordinate to μ it follows from lemma A2 that each Λ_j is subordinate to μ when considered as $U_q(g_0)$ weights. Since in the non-graded case for $\mu \in D^+_{\Lambda_j}$, $\forall j$, the multiplicity of $V_0(\mu+\lambda_i) \subset V(\Lambda) \otimes V_0(\mu)$ is equal to the multiplicity of $\lambda_i \in \Pi(\Lambda)$ [22], it also follows from lemma A2 that the multiplicity is the same in the $\mathbb{Z}_2$-graded decomposition (8).

We now recall the following result due to Parthasary, Ranga-Rao and Varadarajan [23].

Lemma A3. *For $\lambda_i \in \Pi(\Lambda)$ the multiplicity of $V_0(\mu+\lambda_i) \subset V(\Lambda) \otimes V_0(\mu)$ is given by dim $V_{\lambda_i,\mu}(\Lambda)$ where*

$$V_{\lambda_i,\mu}(\Lambda) = \{v \in V_{\lambda_i}(\Lambda) | e_j^{<\mu+\rho,\alpha_j>} v = 0,\, 1 \leq j \leq l\}.$$

Above, $V_{\lambda_i}(\Lambda) \subset V(\Lambda)$ is the space of all vectors of weight λ_i.

It is apparent from lemma A3 that Λ is subordinate to μ if and only if

$$\begin{aligned} e_i^{<\mu+\rho,\alpha_i>} v =& e_i^{<\mu,\alpha_i>+1} v \\ =& 0, \qquad \forall 1 \leq i \leq l, \quad v \in V(\Lambda). \end{aligned}$$

In view of the finite dimensionality of $V(\Lambda)$ there must exist positive integers $m_1, \ldots\ldots, m_l$ such that

$$e_i^{m_i+1} v = 0, \qquad \forall\, v \in V(\Lambda). \tag{10}$$

Now let $\omega_1, \ldots\ldots, \omega_l$ denote the fundamental weights of $U_q(g)$. For $i \neq 0$ they are defined by

$$< \omega_i,\, \alpha_j >= \delta_{ij},$$

while ω_0 is defined by

$$(\omega_0,\, \alpha_i) = \delta_{i0}.$$

Setting

$$\nu_0 = \sum_{i=0}^{l} m_i \omega_i$$

where m_i, $1 \leq i \leq l$ are to satisfy (10) and m_0 is arbitrary, it is clear that Λ is subordinate to ν_0. If $\nu \in D^+$ satisfies $\nu \geq \nu_0$ then Λ is also subordinate to ν_0. We have thus shown

Lemma A4. *Let $\Lambda \in D^+$. There exists $\nu_0 \in D^+$ such that Λ is subordinate to ν whenever $\nu \geq \nu_0$.*

To see that proposition 2 follows from lemmas A1-4, it suffices to observe that the set of typical $\mu \in D^+$ are Zariski dense in D^+. (For a description of the Zariski topolgy on H^* see [24].) Since polynomial functions are continuous in the Zariski topology, we may conclude that proposition 2 holds.

References

1. M. Jimbo, Lett. Math. Phys. **10** (1985), 63; ibid. **11** (1986), 247; V.G. Drinfeld, Proc. I.C.M. (Berkeley, 1986), 798.
2. P.P. Kulish and N. Yu Reshetikhin, Lett. Math. Phys. **18** (1989), 143; M. Chaichian and P.P. Kulish, Phys. Lett. **B236** (1990), 242; A.J. Bracken, M.D. Gould and R.B. Zhang, Mod. Phys. Lett. **A5** (1990), 331; R. Floreanini, V.P. Spiridonov and L. Vinet, Commun. Math. Phys. **137** (1991), 149; R. Floreanini, D.A. Leites and L. Vinet, Lett. Math. Phys. **23** (1991), 127; M. Scheunert, Lett. Math. Phys. **24** (1992), 173; M. Scheunert, J. Math. Phys. **34** (1993), 3780.
3. S.M. Khoroshkin and V.N. Tolstoy, Commun. Math. Phys. **144** (1991), 599.
4. H. Yamane, Proc. Jpn. Acad. **A67** (1991), 108.
5. S. Sarker, J. Phys. A: Math. Gen **23** (1990), L409; ibid. **24** (1991), 1137; F.H.L. Essler and V.E. Korepin, Phys. Rev. **B46** (1992), 9147.
6. S.-M. Fei and R.-H. Yue, J. Phys. A: Math. Gen. **27** (1994), 3715; A. Foerster and M. Karowski, Nucl. Phys. **B408** (1993), 512.
7. F.H.L. Essler, V.E. Korepin and K. Schoutens, Phys. Rev. Lett. **68** (1992), 2960; ibid. **70** (1993), 73.
8. A.J. Bracken, M.D. Gould, J.R. Links and Y.-Z. Zhang, Phys. Rev. Lett. **74** (1995), 2768.
9. L.D. Faddeev, N. Yu Reshetikhin and L.A. Takhtajan, Algebraic Analysis **1** (1989), 129; M.D. Gould, R.B. Zhang and A.J. Bracken, J. Math. Phys. **32** (1991), 2298; J.R. Links and M.D. Gould, Rep. Math. Phys. **31** (1992), 91.
10. R.B. Zhang and M.D. Gould, J. Math. Phys. **32** (1991), 3261.

11. M.L. Nazarov, Lett. Math. Phys. **21** (1991), 123; R.B. Zhang, *The quantum super Yangian and Casimir operators of* $U_q(gl(m|n))$, Lett. Math. Phys. (to appear).
12. R.B. Zhang, J. Math. Phys. **33** (1992), 3918; J.R. Links, M.D. Gould and R.B. Zhang, Rev. Math. Phys. **5** (1993), 345; M.D. Gould, I. Tsohantjis and A.J. Bracken, Rev. Math. Phys. **5** (1993), 533.
13. J. R. Links and R.B. Zhang, J. Math. Phys. **34** (1993), 6016.
14. A.M. Bincer, J. Math. Phys. **24** (1983), 2546; M. Scheunert, J. Math. Phys. **24** (1983), 2681; H.S. Green and P.D. Jarvis, J. Math. Phys. **24** (1983), 1681.
15. M.D. Gould and M. Scheunert, J. Math. Phys. **36** (1995), 435; J.R. Links and M.D. Gould, J. Math. Phys. **36** (1995), 531.
16. A. J. Bracken, G.W. Delius, M.D. Gould and Y.-Z Zhang, J. Phys. A: Math. Gen **27** (1994), 6551; G.W. Delius, M.D. Gould, J.R. Links and Y.-Z. Zhang, *On type I quantum affine superalgebras*, hep-th/9408006, Int. J. Mod. Phys. A (to appear); G.W. Delius, M.D. Gould, J.R. Links and Y.-Z. Zhang, *Solutions of the Yang-Baxter equation with extra non-additive parameters II:* $U_q(gl(m|n))$, hep-th/9411241.
17. J.R. Links and M.D. Gould, Lett. Math. Phys. **26** (1992), 187; J.R. Links, M. Scheunert and M.D. Gould, Lett. Math. Phys. **32** (1994), 231.
18. M.D. Gould, J.R. Links and Y.-Z. Zhang, *Type I quantum superalgebras, q-supertrace and two-variable link polynomials*, University of Queensland and Kyoto University preprint (1995).
19. V.G. Kac, Lect. Notes in Math. **676** (1978), 597.
20. R.B. Zhang, J. Math. Phys. **34** (1993), 1236; J. Phys. A: Math. Gen **26** (1993), 7041.
21. M.D. Gould and J.R. Links, in preparation.
22. B. Kostant, J. Funct. Anal. **20** (1975), 257.
23. K.R. Parthasary, R. Ranga-Rao, and V.S. Varadarajan, Ann. Math. **85** (1967), 383.
24. J.E. Humphreys, *Introduction to Lie algebras and representation theory*, Springer-Verlag, Berlin (1972).